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SMOOTHING THINGS OVER: A THEORETICAL ANALYSIS OF STAGNANT CAPS OF SURFACTANT

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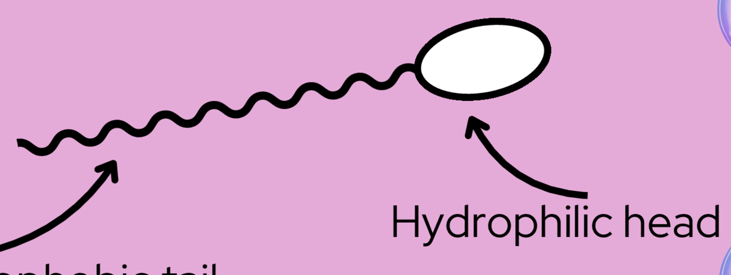
Personal webpage and publications

What are surfactants?

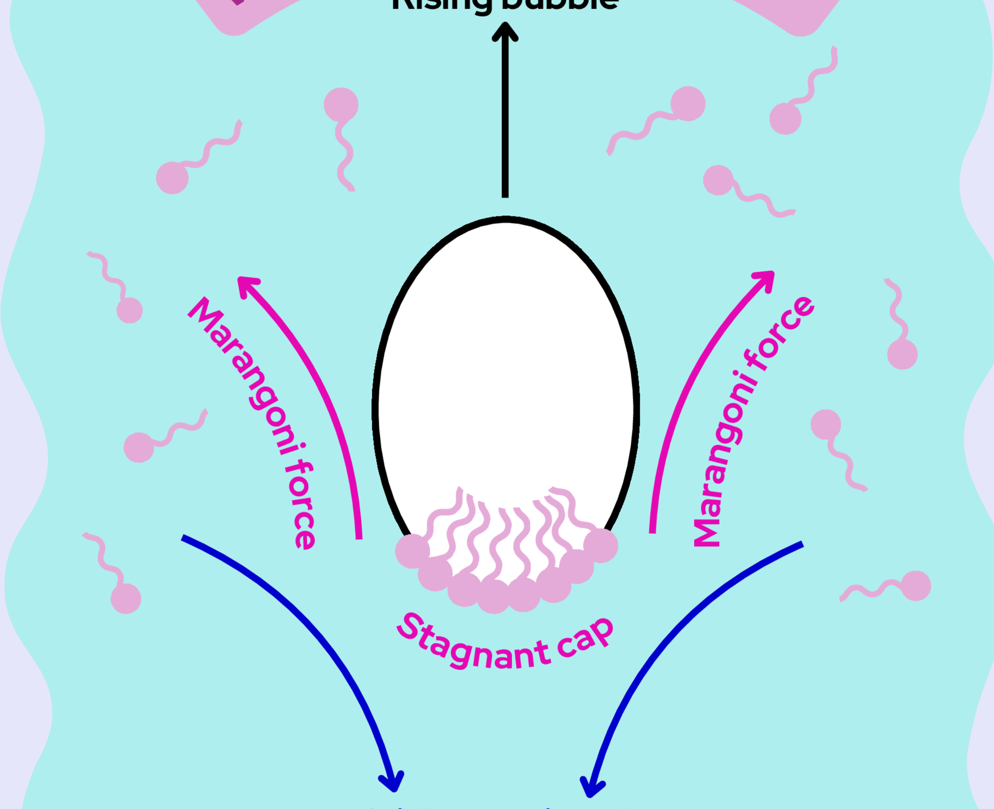
- Surfactants are molecules with a **hydrophobic** (water-hating) tail, and a **hydrophilic** (water-loving) head.



- When present at an interface between two fluids, they **lower the surface tension** there.
- They can be **insoluble** (they stay on the interface), or **soluble** (they can move on and off).
- They appear virtually everywhere, **even in trace amounts**, making them difficult to remove.

Understanding their behaviour in a fluid is therefore crucial

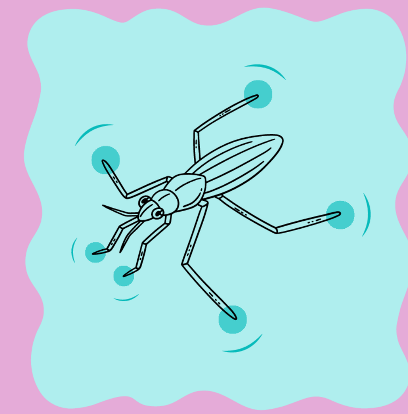
What is a stagnant cap?



- As a gas bubble rises through fluid, surfactant sticks to the surface and becomes densely packed at the rear, **clogging the interface and lowering the surface tension there.**
- This produces a **Marangoni force**, acting from regions of low surface tension to high surface tension, **causing the bubble to rise much more slowly**. This clogged region is called a **stagnant cap.**
- These caps can form at any interface, and can have **adverse effects on the flow.**
- If the surfactant were **spread out uniformly**, the **fluid interface would behave as if it weren't there!**

Surface tension and the Marangoni effect

- **Surface tension** is the tendency of a liquid interface to shrink in order to minimise surface area. Because of it, objects with a higher density than water **can float** without being submerged.



Water striders are insects that manage to "walk" on water because of surface tension.

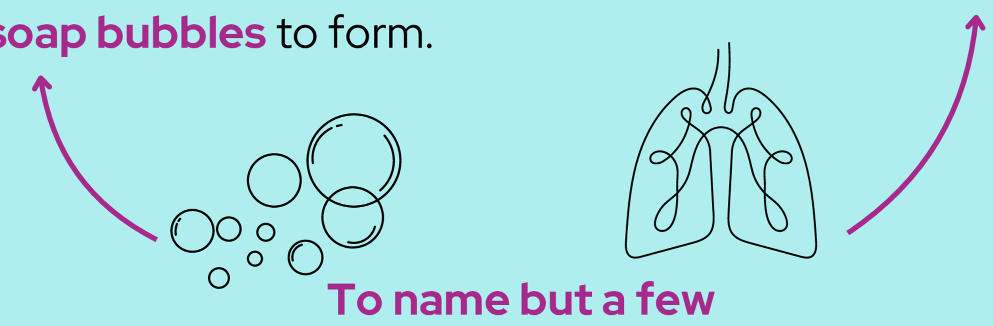
- **The Marangoni effect** occurs when the surface tension is not uniform - a **force** is created, acting in the **direction of low surface tension to high surface tension**, pushing the fluid in that direction.

In a glass of wine, alcohol has a lower surface tension than water and evaporates faster. As it does, ***the surface tension imbalance pulls the wine up the glass*** *- this is the Marangoni effect. Eventually, it drips back down, creating* ***"tears of wine"***, ***visible on this glass.***

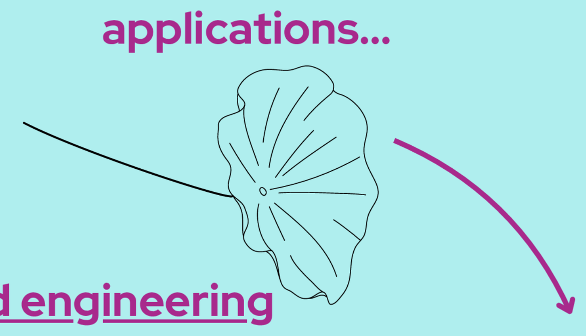
Surfactants are everywhere!

Everyday
Soap and detergents are made up of surfactants - they break down dirt, grease, and bacteria. They also allow **soap bubbles** to form.

Medicine
Premature babies often struggle to inflate their lungs when born - giving them surfactant is **critical in helping them breathe.**



To name but a few applications...



Industry and engineering
When a **water droplet rolls off a lotus leaf** or a raincoat, this is called a superhydrophobic surface. They can be used in applications such as **self-cleaning surfaces, faster drug delivery, and laptop cooling systems.** The presence of surfactants **destroys** the water-repelling properties that underpin these applications.

The questions we seek to address

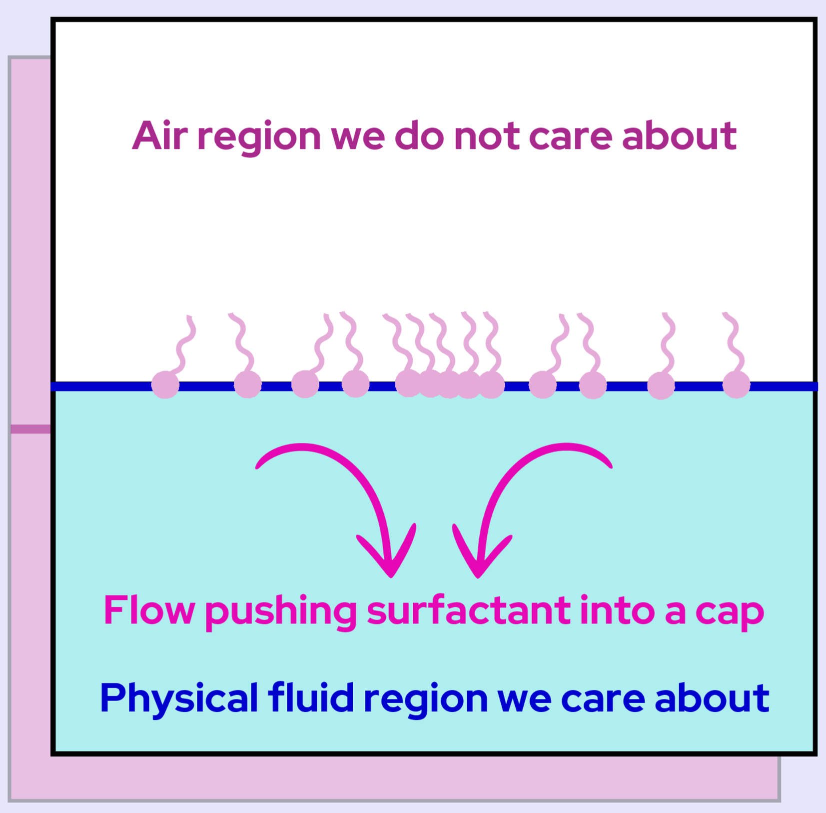
How does surfactant affect a flow?

Can we control sharp gradients in the surface tension?

How can we capture these phenomena mathematically?

We investigate a class of problems where:

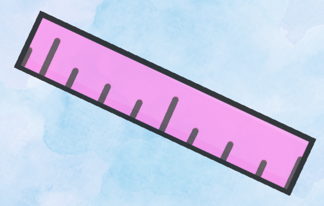
- **Surfactant occupies an interface between a liquid and an air region.**
- **An imposed flow is forcing the surfactant towards a cap formation.**
- **Our goal is to spread the surfactant out.**



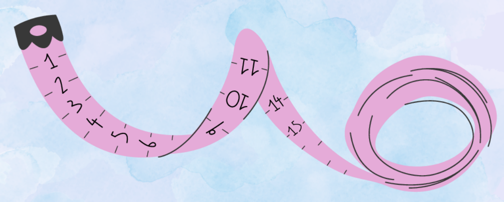
Multiphysics problems such as this one involve **many governing equations** for the flow and surfactant concentration that must be satistfied.

By using imaginary numbers!

If we want to define the square root of a negative number, we do so by introducing $\sqrt{-1} = \mathrm{i}$, which we call an **imaginary number.** Imaginary numbers can be used to construct the **complex plane**, $z = x + \mathrm{i}y$, where (x, y) represent the usual real 2D coordinates.



Think of the **real numbers as a ruler**, the directions in which you can measure are **constrained.**



Complex numbers are like a measuring tape, they give you the **geometric flexibility** required to solve your real-world problem.

Complex numbers may seem abnormal to use, however **they are the natural choice** for solving many physical problems. **The quickest route in the real world is often via the complex plane!**

But by **embedding our real-world problem in the complex plane**, we are able to **couple all governing equations** and reduce them to a **single ordinary differential equation**, which we can **solve explicitly!**

With an explicit solution, we can find which **physical parameters** we need to modify in order to **control and spread the surfactant out.**

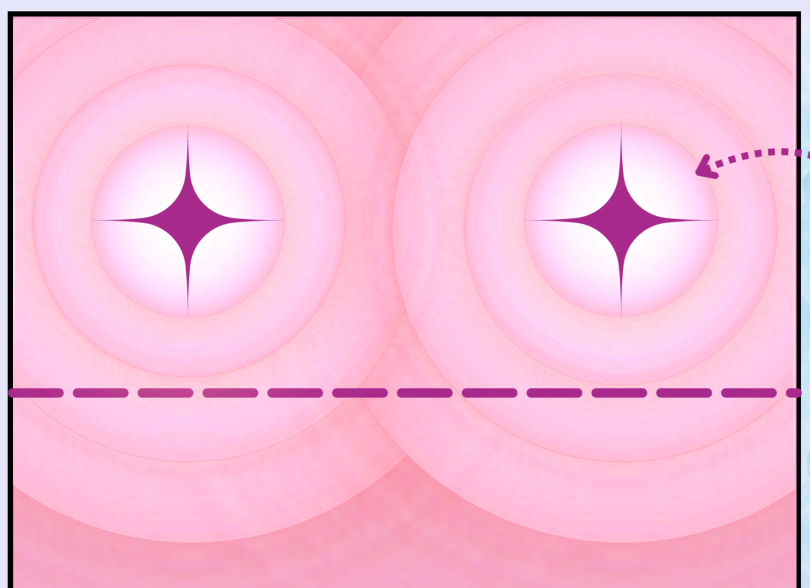
Why do we need singularities?

- Points at which a function is **not defined** are called **singularities**.
- They might seem like a **mathematical disaster**, however they are essential for making interesting physical things happen!

Think about a **swirling vortex** in your sink - your plughole is like the singularity, **forcing the interesting fluid motion.**

- Their location in a non-physical region can **drive the flow dynamics** in the physical region we are interested in.
- We must therefore consider the full problem **in this hidden plane.**

The hidden complex plane!



Singularities in the unphysical upper half-plane **drive the dynamics of the flow** in the real-world fluid.

When we **consider the hidden complex plane** behind our real-world problem, we find **singularities** outside of the fluid region that are **driving the dynamics of the flow**. Their location tells us **how the fluid and surfactant are behaving**, and **how to smooth out the cap.**

With our solution, sharp gradients in the surfactant profile are explicitly smoothed out!

We have successfully used this approach to **explicitly model** surfactant problems on **flat and circular interfaces**, for both **soluble and insoluble surfactant**, subjected to **different physical forcings.**