Understanding rotational flows via mathematical modelling

C. White, N.R. McDonald
Department of Mathematics, University College London

UKRI EPSRC

UCL

What is a Vortex?

A vortex is a region of fluid that experiences local rotational motion such as water spiraling down a sink or swirling in ocean whirlpools.

Tropical storms on Earth in the last 50 years have caused

$1.4 Trillion USD in economic damages,

779,324 fatalities worldwide.

It is estimated Hurricane Melissa (2025) (A) caused **$8.8 Billion USD** in economic damage to Jamaica.

Climate Change is Affecting Tropical Storms

Greater wind speeds and flooding are being observed due to warmer atmosphere and raising sea levels.

Storms lasting over 300 years!

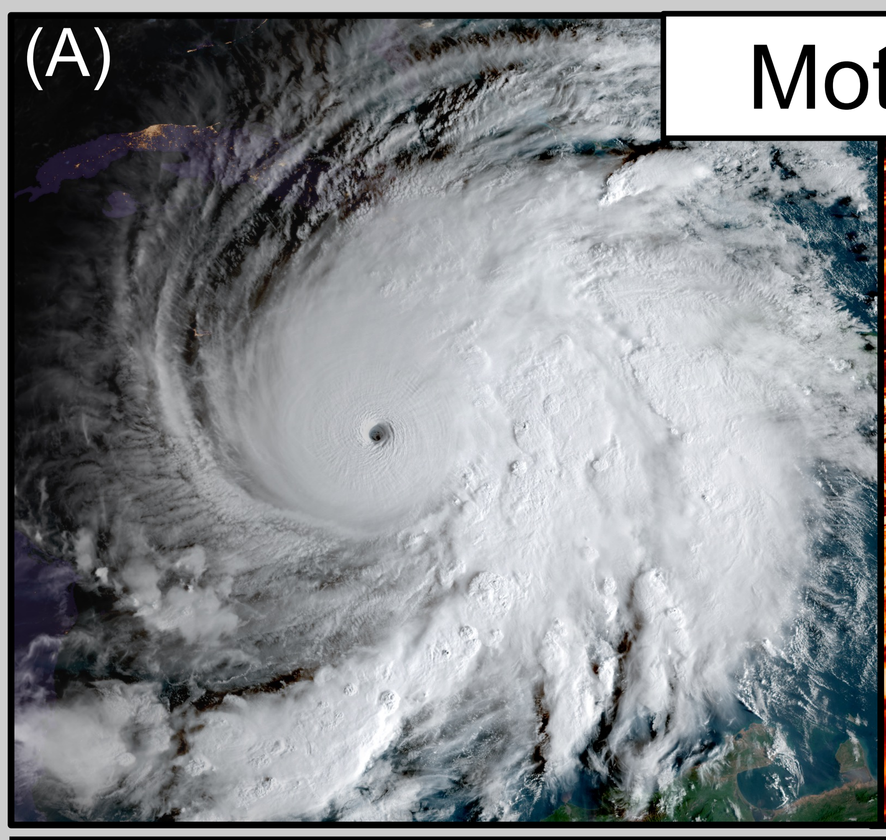
- Jupiter's Great Red Spot is a storm that may have existed for over 300 years.
- Jupiter's storms with geometrical form (B) and have existed for decades, if not centuries.

Vortex Equilibria

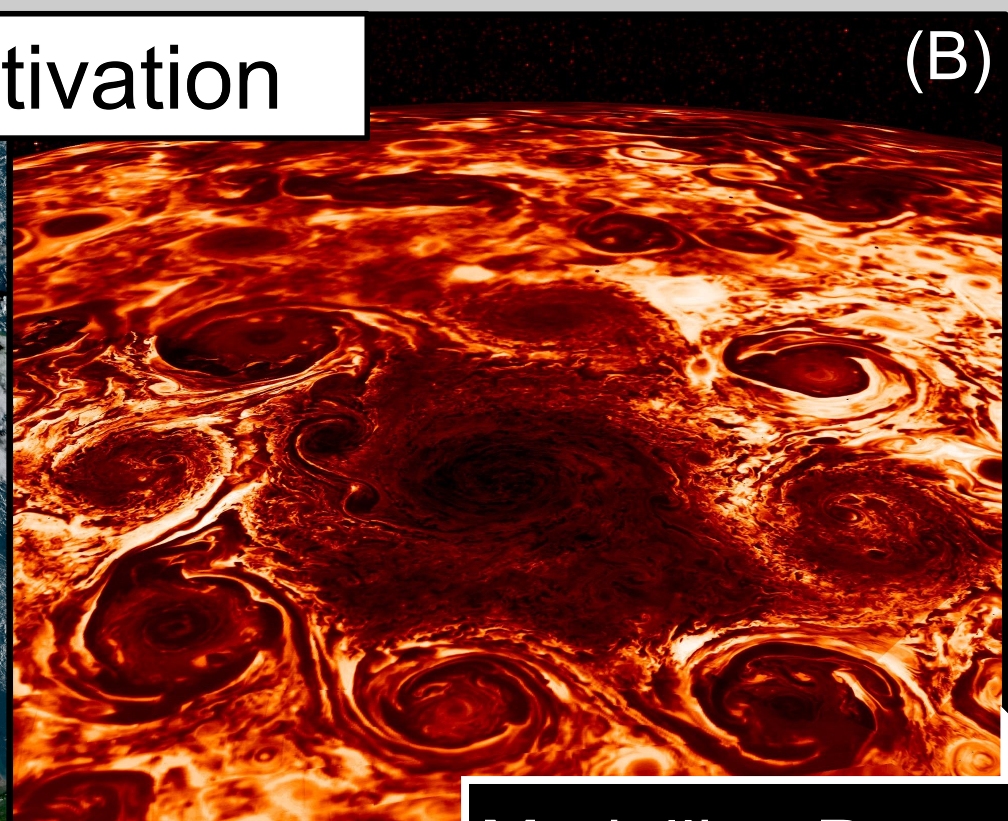
- The long-lasting nature of such flows suggests they are in stable equilibrium.
- Equilibrium – same shape over time, even if they translate or rotate with constant speed.
- Studying equilibria for any dynamical system matters because they reveal a system's long-term behavior.

Motivation

(A)

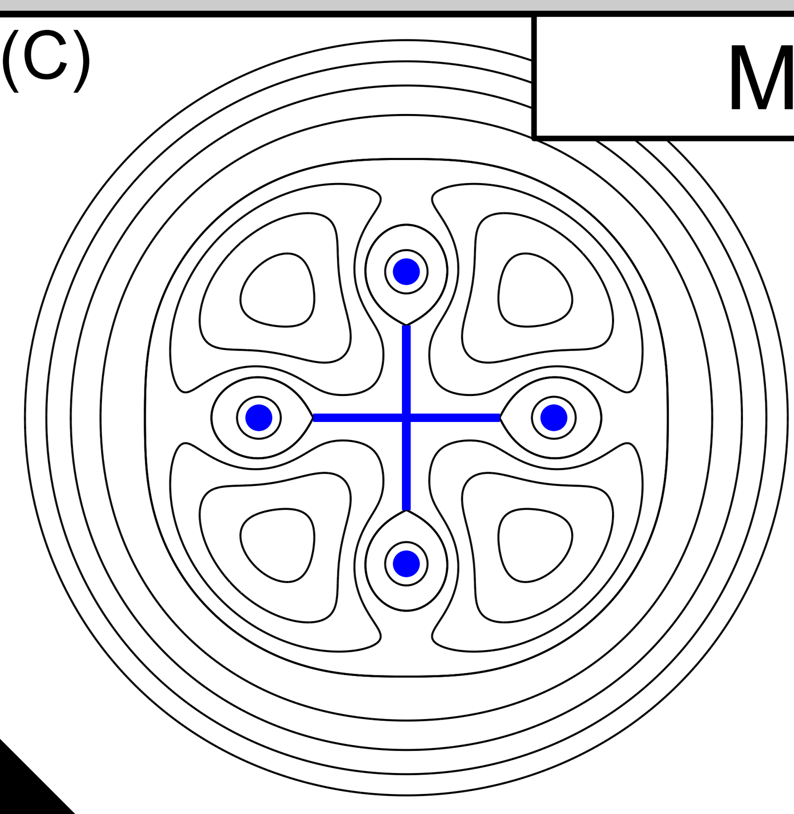


(B)

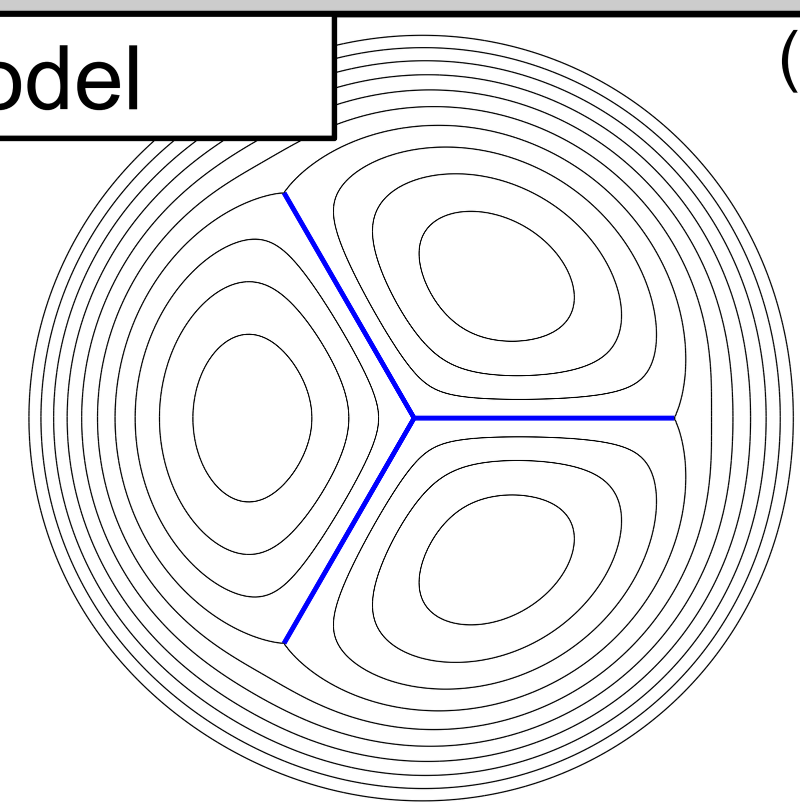


Model

(C)



(D)



Modelling Process

Assumptions

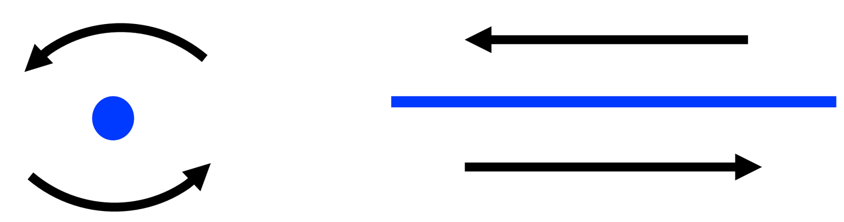
1. Two-dimensional flow
Motion mostly sideways & small change in height so study horizontal slice.

2. Incompressible fluid
Compressible fluids **squash** or **expand**.

2. Inviscid fluid (ignore viscosity)
Honey has **high viscosity** as it is **sticky**.

4. **Irrotational flow** + **compact rotation**
The **fluid** doesn't spin locally; the rotation is concentrated into **points** or **lines** e.g. as in (C) and (D) above so the **fluid** spins globally.

The regions of **compact rotation** drive the **flow** to generate the geometric flow patterns.



Complex Analysis

Point vortex models give quick estimates for moving vortices – useful for the Met Office!

A **point vortex** drives the **flow** around it in concentric circles. A **vortex sheet** drives the **flow** either side of it in opposite directions.

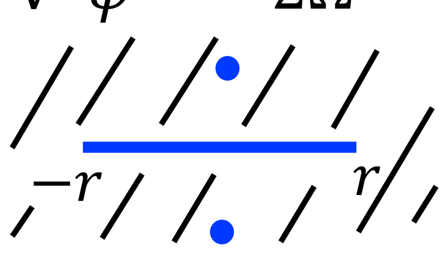
The Research Problem

Can we find new rotating equilibrium configurations involving vortex sheets and point vortices using conformal mapping methods?

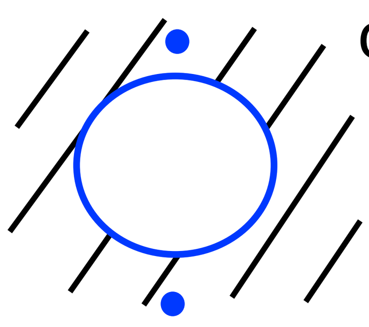
Method and Key Findings

In a frame rotating with constant angular velocity, Ω:

$\nabla^2\psi = -2\Omega$



transform problem to simpler geometry



A given **sheet** strength can produce different steady flow patterns e.g. multiple r values.

Point vortices at **sheet** tips vs. between **sheets** changes system behaviour,

e.g.

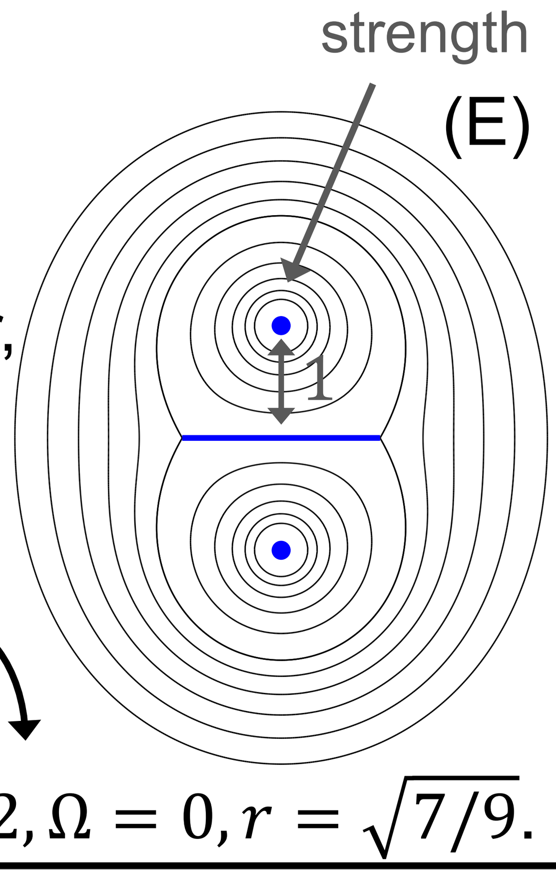
(C) & (D) can only rotate but (E) is stationary if

(sheet strength)

$\gamma = -1/2, \Omega = 0, r = \sqrt{7/9}$.

fixed strength

(E)



Achievements

Computational simulations are costly and often conceal underlying mechanisms.

This work provides new closed form mathematical descriptions:

- Useful for understanding theory.
- Accessible to non-specialists.
- Almost no computational cost.
- Provides a benchmark for larger simulations.

Image sources: (A) NOAA, (B) NASA

C. White, N.R. McDonald (2025) Exact solutions for vortex equilibria by conformal mapping. *Proc. R. Soc. A*, **481**: 20250362